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# MODELING AND OPTIMIZATION OF AN AUTOMATIC POWER CONTROL SYSTEM FOR THE MILLING PROCESS

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***Abstract*** **—**The article addresses the scientific and applied problem of developing automatic control systems for stabilizing milling machine cutting power under changing operating conditions. Maintaining a constant cutting power level is essential for improving machining quality, increasing productivity, reducing energy consumption, and extending tool life.
A synthesis approach based on fractional-integral controllers with an increased astatism order is proposed. A specialized optimization criterion was developed to ensure fast transient response while limiting overshoot within predefined bounds. Unlike conventional mean-square-error criteria, the proposed functional accounts for the position of the control error relative to the admissible region, providing smoother transient behavior.
Optimal controller parameters were determined using genetic algorithms implemented in MATLAB. The obtained results enabled the synthesis of fractional-order control systems with improved dynamic performance compared to classical controllers. Simulation studies demonstrated effective disturbance rejection and rapid stabilization under sudden load changes. For a 50% increase in cutting power demand, the transient deviation was reduced from approximately 200 W to 19–39 W, while the settling time remained below 0.02 s.
***Keywords* —** cutting power, milling machine, automatic control system, fractional-integral controller, fractional order

# ФРЕЗЕРЛЕУ ПРОЦЕСІНІҢ ҚУАТЫН АВТОМАТТЫ РЕТТЕУ ЖҮЙЕСІН МОДЕЛЬДЕУ ЖӘНЕ ОҢТАЙЛАНДЫРУ

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***Аңдатпа*** — Жұмыста астатизм дәрежесі жоғарылаған бөлшек-интегралдық реттегіштерге негізделген синтез әдісі ұсынылған. Өтпелі процестің жоғары жылдамдығын қамтамасыз ете отырып, артық реттеуді алдын ала белгіленген шектерде шектеуге мүмкіндік беретін арнайы оңтайландыру критерийі әзірленді. Дәстүрлі орташа квадраттық қателік критерийлерінен айырмашылығы, ұсынылған функционал басқару қателігінің рұқсат етілген аймаққа қатысты орналасуын ескереді, бұл өтпелі процестердің неғұрлым бірқалыпты өтуін қамтамасыз етеді. Реттегіштің оңтайлы параметрлері MATLAB ортасында жүзеге асырылған генетикалық алгоритмдер көмегімен анықталды. Алынған нәтижелер классикалық реттегіштермен салыстырғанда динамикалық сипаттамалары жақсартылған бөлшек ретті басқару жүйелерін синтездеуге мүмкіндік берді. Имитациялық зерттеулер жүктеменің күрт өзгеруі

жағдайында сыртқы әсерлерді тиімді өтеуді және жүйенің жылдам тұрақтануын көрсетті. Кесу қуатына сұраныс 50%-ға артқан кезде өтпелі ауытқу шамамен 200 Вт-тан 19–39 Вт-қа дейін төмендегені анықталды, ал орнығу уақыты 0,02 секундтан аспады.
***Кілт сөздер*** —. кесу қуаты, фрезерлік станок, автоматты басқару жүйесі, бөлшек-интегралдық реттегіш, бөлшек ретті жүйе.

# МОДЕЛИРОВАНИЕ И ОПТИМИЗАЦИЯ СИСТЕМЫ АВТОМАТИЧЕСКОГО УПРАВЛЕНИЯ МОЩНОСТЬЮ ПРОЦЕССА ФРЕЗЕРОВАНИЯ

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***Аннотация*** — Предложен подход к синтезу систем управления на основе дробно-интегральных регуляторов с повышенным порядком астатизма. Разработан специализированный критерий оптимизации, обеспечивающий высокое быстродействие переходного процесса при ограничении перерегулирования в заданных пределах. В отличие от традиционных критериев среднеквадратической ошибки, предложенный функционал учитывает положение ошибки управления относительно допустимой области, что обеспечивает более плавное протекание переходных процессов. Оптимальные параметры регуляторов были определены с использованием генетических алгоритмов, реализованных в среде MATLAB. Полученные результаты позволили синтезировать системы управления дробного порядка с улучшенными динамическими характеристиками по сравнению с классическими регуляторами. Результаты моделирования показали эффективное подавление возмущений и быстрое восстановление режима при резких изменениях нагрузки. При увеличении требуемой мощности резания на 50 % величина переходного отклонения была снижена приблизительно с 200 Вт до 19–39 Вт, а время установления не превышало 0,02 с.

***Ключевые слова:*** мощность резания, фрезерный станок, автоматическая система управления, дробно-интегральный регулятор, система дробного порядка.

## I. INTRODUCTION

The current stage of mechanical engineering development is characterized by rapidly increasing requirements for production efficiency, product quality, automation level, and energy efficiency of technological equipment. Under conditions of global competition, industrial enterprises are forced to continuously improve labor productivity, reduce material and energy consumption, and ensure consistently high machining accuracy. Achieving these objectives largely depends on the sophistication of automatic control systems, which must not only maintain specified operating modes but also adapt to changing production conditions.

Milling occupies a leading position among modern machining processes. Owing to its versatility, high productivity, and capability to manufacture parts with complex geometries, milling operations are widely used in aerospace, automotive, shipbuilding, energy, and many other industrial sectors. Consequently, improving the efficiency of milling machine operation remains an important research area attracting considerable attention from specialists in automation, mechatronics, and electric drive systems.

The efficiency of the milling process is largely determined by the stability of cutting conditions. During machining, the cutting tool and machine drive are continuously subjected to variable loads caused by the technological characteristics of the process. The main factors affecting cutting power include variations in the physical and mechanical properties of the workpiece material, fluctuations in machining allowance, changes in cutting depth and width, feed rate variations, vibration phenomena, thermal effects, and progressive tool wear. In practical manufacturing environments, all these factors act simultaneously, creating a complex combination of internal and external disturbances that significantly influence the energy characteristics of the cutting process.

Variations in cutting power directly affect the quality of the machined surface and the dimensional accuracy of the workpiece. Excessive cutting power may lead to drive overload, increased heat generation, accelerated tool wear, and deterioration of surface finish. Conversely, insufficient loading often results in inefficient equipment utilization, reduced productivity, and longer machining cycles. Therefore, maintaining cutting power at a prescribed level represents one of the most important objectives of modern automation systems for metal-cutting operations.

Traditionally, automatic control systems for machine-tool drives have focused on stabilizing spindle speed or feed rate. Such systems provide the required operating conditions for the drive and maintain specified kinematic parameters of the machining process. However, practical experience shows that even with perfect spindle-speed stabilization, cutting power may vary considerably under the influence of random load disturbances. As a result, additional dynamic stresses arise in machine components, machining accuracy deteriorates, and operating costs increase.

For this reason, increasing attention has recently been devoted to the development of control systems aimed directly at stabilizing the energy parameters of machining processes. One of the most promising approaches involves adjusting the tool feed rate according to the current value of cutting power. In this case, the control system automatically compensates for load variations, maintaining an optimal operating mode regardless of changes in material properties or other external factors. The implementation of such a principle makes it possible to increase productivity, ensure more uniform loading of the electric drive, and significantly improve the economic performance of manufacturing operations.

At the same time, the development of highly efficient cutting-power stabilization systems is associated with a number of challenging scientific and technical problems. The control object exhibits pronounced nonlinear behavior, time-varying parameters, and random disturbances. Moreover, different control quality indicators often impose conflicting requirements. For example, increasing system speed frequently leads to greater overshoot, whereas reducing steady-state error may decrease stability margins or increase settling time. Consequently, the search for new controller structures and parameter optimization methods remains an important direction of contemporary research.

Among the promising approaches for improving control quality, fractional-integral controllers have attracted considerable attention. Unlike conventional PI, PID, and related controllers, they employ the mathematical apparatus of fractional calculus, allowing integration and differentiation of non-integer order. This approach provides additional degrees of freedom for shaping system dynamics and offers significantly greater flexibility in controller tuning. Owing to these additional parameters, fractional-integral controllers are capable of providing high control accuracy, enhanced disturbance rejection, and an optimal balance between response speed and damping characteristics.

Particular interest is associated with systems possessing a fractional order of astatism. Increasing the astatism order makes it possible to significantly reduce dynamic errors and improve tracking performance under varying reference inputs. However, a higher astatism order also increases controller complexity, necessitating dedicated studies aimed at determining optimal parameter values. Another important challenge involves the development of optimization criteria that account not only for the magnitude of the control error but also for transient response characteristics, overshoot level, and settling speed.

Modern global optimization techniques, particularly genetic algorithms, offer substantial potential for addressing these challenges. Evolutionary optimization methods enable the efficient determination of optimal controller parameters in multidimensional search spaces while avoiding local extrema commonly encountered in gradient-based approaches. Consequently, it becomes possible to synthesize control systems with predetermined performance characteristics and high operational efficiency over a wide range of operating conditions.

Therefore, the relevance of this research is determined by the need to improve the efficiency of machining processes through the development of advanced cutting-power stabilization systems for milling machines. The synthesis of closed-loop control systems based on fractional-integral controllers and modern optimization algorithms can provide high levels of accuracy, dynamic performance, and energy efficiency. The obtained results may be applied in the design of next-generation machine-tool drives, automated manufacturing systems, and robotic production complexes, thereby contributing to the further advancement of modern mechanical engineering and digital manufacturing technologies.

## II. LITERATURE REVIEW

Improving the efficiency of machining processes on metal-cutting machine tools is one of the key directions in the development of modern mechanical engineering. Under conditions of continuously increasing requirements for product quality, production productivity, and energy efficiency of technological processes, the improvement of automatic control systems for cutting conditions becomes particularly important. The ability to ensure stable machining accuracy, rational utilization of equipment, and minimization of production costs largely depends on the effectiveness of such control systems. Modern manufacturing facilities operate in a highly competitive environment, forcing enterprises to continuously seek new approaches to increasing productivity and reducing production costs. One of the most promising ways to achieve these objectives is the implementation of intelligent automatic control systems for technological processes.

Among the numerous parameters characterizing the machining process, cutting power occupies a special place. This parameter directly reflects the energy state of the technological process and integrally accounts for the influence of numerous factors arising during machine operation. Cutting power depends on the physical and mechanical properties of the workpiece material, the geometric characteristics of the cutting tool, cutting conditions, tool wear level, rigidity of the technological system, and many other factors. Due to its high informativeness, this parameter is widely used as a criterion for evaluating machining efficiency and as a control variable in modern automated systems.

Maintaining cutting power at a specified level enables the rational use of energy resources and ensures maximum utilization of technological equipment. Under conditions of stable cutting power, a uniform load distribution on the electric drive is achieved, mechanical and thermal overloads are reduced, and the operating conditions of the cutting tool are improved. As a result, the quality of the machined surface is enhanced, the amount of defective products is reduced, equipment service life is extended, and manufacturing costs are lowered. Furthermore, cutting-power stabilization creates the prerequisites for implementing adaptive control systems capable of automatically adjusting operating modes according to the current state of the technological process.

The necessity of cutting-power stabilization arises from the fact that real machining processes are accompanied by numerous external and internal disturbances. These include variations in machining allowance, non-uniformity of the workpiece material structure and hardness, changes in tool geometry caused by wear, vibration phenomena, thermal effects, and other factors. Under the influence of these disturbances, cutting power may significantly deviate from its nominal value, resulting in reduced machining quality and decreased equipment efficiency. Therefore, the development of automatic control systems capable of rapidly compensating for such disturbances represents an important scientific and technical challenge.

Traditional approaches to the design of automatic control systems for metal-cutting machine tools are primarily based on classical controllers, such as proportional, integral, differential, and their combined forms. Despite their widespread use and relative simplicity, these controllers do not always provide the required control quality when applied to complex nonlinear objects with time-varying parameters. In many cases, it becomes necessary to satisfy conflicting requirements related to response speed, accuracy, stability, and overshoot limitation, which significantly complicates the controller tuning process.

For this reason, fractional-order control methods have gained considerable attention in recent years. These methods are based on the mathematical apparatus of fractional calculus. Unlike conventional approaches, fractional-order controllers enable integration and differentiation of non-integer order, thereby providing additional degrees of freedom in the synthesis of automatic control systems. This makes it possible to achieve more flexible shaping of dynamic and static characteristics of closed-loop systems and to obtain performance indicators that are difficult to achieve using traditional controllers.

Particular interest in fractional-order control methods is explained by their ability to adequately describe processes characterized by distributed parameters, complex dynamics, and pronounced inertial properties. Many machining processes exhibit precisely these features, making the application of fractional-order models and controllers highly justified. Numerous studies have demonstrated that fractional-integral and fractional-derivative controllers can significantly improve control quality, reduce dynamic errors, and increase system robustness against external disturbances.

Thus, the development of fractional-order control methods opens new opportunities for improving automatic cutting-power stabilization systems for metal-cutting machine tools. The application of fractional-integral controllers enables the formation of optimal dynamic and static characteristics of closed-loop systems while ensuring high accuracy in maintaining technological parameters, increasing equipment productivity, and reducing energy consumption. Therefore, research aimed at developing and improving methods for the synthesis of such control systems is highly relevant from both scientific and practical perspectives.

In [1], the authors considered a microprocessor-based control system for an AC electric drive with regulation of a technological parameter. The study demonstrated the feasibility of developing control systems focused not only on maintaining the kinematic coordinates of an electric drive but also on directly regulating technological process variables. The obtained results confirmed the prospects of using technological parameters as control coordinates. However, issues related to achieving optimal dynamic performance and the application of fractional-order controllers were not addressed.

The theoretical foundations for applying fractional mathematical tools were presented in [2], where the possibility of approximating transient thermal processes in complex geometric structures using fractional derivatives was investigated. The obtained results confirmed the effectiveness of fractional-order models in describing complex physical processes characterized by distributed parameters and pronounced inertial properties.

Practical aspects of implementing control systems based on fractional calculus were considered in [3]. The authors performed the synthesis and practical implementation of automatic control systems employing discrete fractional-integral-derivative controllers. It was demonstrated that the use of fractional operators expands controller tuning capabilities and enables the achievement of desired transient response characteristics.

Further development of this research direction was presented in [4], which focused on the modeling and identification of systems containing fractional integral and differential elements. The proposed approaches established a methodological basis for constructing fractional-order mathematical models and synthesizing corresponding control systems.

Fundamental principles of fractional dynamics and control theory were systematized in [5]. The authors presented the mathematical foundations of fractional calculus, methods for analyzing fractional-order systems, and principles for designing controllers that provide additional degrees of freedom in shaping the dynamic characteristics of closed-loop systems.

The widespread application of fractional-order models in modern engineering was confirmed in the review paper [6], devoted to the modeling of supercapacitors, rechargeable batteries, and fuel cells. The authors showed that fractional models provide a more accurate description of energy storage and conversion processes compared with conventional integer-order models.

A significant contribution to the development of fractional-order system synthesis methods was made in [7]. The authors proposed a methodology for designing automatic control systems based on standard fractional Butterworth forms and fractional binomial forms. The obtained results demonstrated the possibility of purposefully shaping the required frequency and dynamic characteristics of control systems through the selection of the fractional integration order.

The practical effectiveness of fractional-order controllers in technological processes was demonstrated in [8], where an ore grinding circuit was controlled using fractional-order controllers. The research results confirmed improved control quality and disturbance rejection capabilities compared with conventional PID controllers.

A modern trend in the development of fractional-order control is the integration of fractional controllers with adaptive algorithms. Thus, [9] proposed fractional PI–PIμD controllers with neural-network adaptation for BLDC motor drive systems. The obtained results demonstrated improved control accuracy and enhanced dynamic performance through the application of fractional-order integration and differentiation.

In [10], PIλDμ controllers were investigated for improving human–machine interaction in complex technical systems. The authors analyzed the influence of fractional-order controllers on control system performance and demonstrated their potential for application in complex engineering objects.

Practical aspects of using fractional-order controllers to optimize dynamic processes were considered in [11], where a control system for electromagnetic brakes of an internal combustion engine test bench employing fractional ID controllers was investigated. The performed analysis confirmed the effectiveness of fractional-order controllers in improving control quality and enhancing the dynamic performance of electromechanical systems.

The conducted analysis of the available literature indicates significant progress in the development of fractional-order control theory, methods for mathematical modeling of fractional-order systems, and the practical application of fractional-integral and fractional-derivative controllers in various engineering systems. Over the past decades, a substantial scientific foundation has been established, encompassing both the mathematical principles of fractional calculus and the applied aspects of its implementation in automatic control systems. Numerous theoretical and experimental studies have convincingly demonstrated that fractional-order controllers extend the possibilities for shaping the dynamic characteristics of closed-loop systems, improve control accuracy, and provide superior transient response performance compared with conventional integer-order controllers.

The analysis of scientific publications shows that fractional-order controllers are most actively employed in electric drive systems, power engineering installations, robotic complexes, electromechanical systems, thermal engineering facilities, chemical process control systems, and objects with distributed parameters. Considerable attention has been devoted to the synthesis of fractional $PI^{\lambda}$, $PD^{\mu}$, and $PI^{\lambda}D^{\mu}$ controllers, the investigation of their frequency-domain characteristics, stability analysis, and the development of parameter optimization methods. Numerous researchers have demonstrated that fractional-order integration and differentiation provide additional degrees of freedom during controller design, enabling higher control quality in systems characterized by complex dynamic behavior.

At the same time, the performed analysis revealed that the vast majority of existing studies are focused either on general theoretical aspects of fractional-order control or on the application of fractional controllers to specific classes of electromechanical systems. Considerably less attention has been paid to technological objects in which the controlled variables are directly related to manufacturing process parameters. In particular, comprehensive studies devoted to the development of automatic cutting-power stabilization systems for metal-cutting machine tools using fractional-order controllers are practically absent in the available scientific literature.

A distinctive feature of the cutting process is that cutting power represents an integral indicator of the technological process state while simultaneously reflecting the influence of numerous internal and external factors. It depends on the properties of the workpiece material, cutting conditions, tool geometry, tool wear level, machining allowance, rigidity of the technological system, and many other parameters. Therefore, stabilizing cutting power makes it possible to maintain optimal operating conditions and ensure high technical and economic performance of manufacturing equipment. Despite its importance, the use of cutting power as the primary controlled variable of the machining process has received insufficient attention in contemporary scientific research.

An analysis of existing approaches indicates that most automatic control systems for metal-cutting machine tools are primarily oriented toward stabilizing spindle rotational speed or feed rate. Such systems ensure the maintenance of kinematic process parameters but do not always guarantee the stability of the cutting energy regime. As a result, significant power fluctuations may occur under varying load conditions, negatively affecting machining quality, equipment productivity, and cutting-tool life. Consequently, there is a need for specialized automatic control systems directly focused on cutting-power stabilization.

Another insufficiently investigated issue concerns the selection of the optimal astatism order of closed-loop systems. It is well known that increasing the astatism order reduces steady-state and dynamic errors, improves tracking accuracy, and enhances disturbance rejection capabilities. However, a higher astatism order also increases controller complexity and alters system dynamic behavior. Existing studies do not provide generalized recommendations regarding the selection of the fractional astatism order for cutting-power stabilization systems, nor do they sufficiently investigate its influence on transient response quality indicators.

A separate scientific challenge is associated with the development of optimization criteria for the synthesis of fractional-order control systems. Most existing approaches are based on integral quadratic performance indices or their modifications. Although such criteria effectively minimize the mean-square control error, they do not always ensure an optimal compromise between response speed, overshoot, and oscillatory behavior. For technological systems in which even a short-term excess of the permissible power level may result in reduced machining quality or equipment overload, there is a need for specialized optimization criteria capable of accounting for the specific features of the technological process.

Further investigation is also required regarding the determination of optimal parameters for fractional-integral controllers. The number of adjustable parameters in such controllers is significantly greater than that of conventional PI and PID controllers. On the one hand, this creates additional opportunities for improving control performance; on the other hand, it substantially complicates the parameter optimization problem. Therefore, the development of efficient tuning algorithms capable of determining optimal controller parameters over a wide range of operating conditions remains an important research objective.

Particular attention should be given to the problem of simultaneously achieving high response speed, minimal overshoot, and low dynamic errors under abrupt load variations. For cutting-power stabilization systems, this requirement is of fundamental importance because any delay or excessive fluctuation of the controlled variable may negatively affect the machining process. Consequently, there is a need to develop new synthesis methods for fractional-order control systems that provide an optimal compromise among all major control quality indicators.

Therefore, despite the considerable achievements in the field of fractional-order control, the problem of synthesizing closed-loop cutting-power stabilization systems for metal-cutting machine tools using fractional-integral controllers remains insufficiently investigated. Future research should focus on developing methods for selecting the optimal astatism order, creating specialized optimization criteria for transient processes, determining the tuning laws of fractional-order controllers, and ensuring high control quality under intensive load variations. Addressing these scientific and technical challenges constitutes the primary objective of the present study.

*The objective of this paper* is to develop a scientifically grounded methodology for the synthesis of closed-loop cutting-power stabilization systems for milling machines based on fractional-integral controllers capable of providing high control quality under variable operating conditions of technological equipment. Particular attention is focused on the formation of control system structures and parameter sets that simultaneously ensure high response speed, minimal overshoot, low steady-state and dynamic errors, and enhanced robustness against external and internal disturbances characteristic of machining processes.

To achieve this objective, a comprehensive set of theoretical and applied studies was carried out aimed at improving automatic cutting-power control methods. In particular, the operational features of cutting-power stabilization systems were analyzed, and the main factors affecting control quality under varying load conditions were identified. The feasibility of employing fractional-integral controllers as an effective means of improving control performance was substantiated due to the additional degrees of freedom provided by the mathematical apparatus of fractional calculus. Furthermore, a specialized optimization criterion for transient processes was formulated, aimed at achieving the maximum possible system response speed while simultaneously limiting overshoot and dynamic deviations of the controlled variable.

Within the framework of the study, optimal parameters of the closed-loop control system were determined for various astatism orders, the corresponding parameter variation patterns were established, and the influence of controller parameters on system dynamic characteristics was evaluated. Based on the obtained results, fractional-integral controllers were synthesized to improve cutting-power stabilization accuracy under abrupt load changes and random disturbances. In addition, transient responses were investigated, control quality indicators were assessed, and rational system parameters ensuring effective maintenance of the specified cutting-power level were determined. The obtained results are aimed at increasing the productivity of milling machines, improving machining quality, and reducing energy consumption in modern automated manufacturing systems.

## III. MATERIALS AND METHODS

### Methodological Approach to the Evaluation of Transient Response Quality in Astatic Control Systems

Let us assume that the inner loop of the closed-loop control system can be described by the following transfer function. Such an assumption is justified from the standpoint of further analysis of the system's dynamic properties and controller synthesis, since it allows the principal operating characteristics of the system to be represented with sufficient accuracy over the entire range of operating conditions. The transfer function of the inner loop characterizes the relationship between the input control signal and the output variable of the system while taking into account the dynamic properties of the actuating mechanisms, electric drive, and other components of the controlled object.

The proposed mathematical representation constitutes a generalized model that makes it possible to investigate the processes of control signal formation, evaluate transient-response quality indicators, and determine the influence of system parameters on stability and response speed. The use of such a model is a common approach in the analysis and synthesis of automatic control systems because it enables the application of well-established methods of control theory to obtain analytical relationships and perform numerical investigations.

It is assumed that the parameters of the transfer function are determined on the basis of the characteristics of the actual controlled object and reflect its inertial properties, gain, and energy-conversion features during system operation. This approach makes it possible to subsequently synthesize controllers and optimize their parameters in accordance with specified requirements regarding control accuracy, response speed, and transient-process quality. Therefore, the inner loop of the closed-loop control system will be described by the following transfer function:

$$H_O(s)=\frac{1}{T_\nu p+1}\frac{k}{a_2 p^{1+\mu}+a_1 p^{\mu}+1}, \qquad (1)$$

where $T_\nu$ denotes the uncompensated small time constant that characterizes the high-frequency dynamics neglected during the compensation procedure.

For the subsequent synthesis of the automatic control system, the desired dynamic properties will be specified by defining an appropriate open-loop transfer function $H_{opt}(p)$. The use of a desired transfer function is one of the most common approaches in control theory, since it makes it possible to determine the required performance characteristics of the future system already at the design stage, including response speed, control accuracy, stability, and the nature of transient processes. In this approach, the structure and parameters of the controller are selected in such a way that the actual system behavior is as close as possible to the desired model.

Let the desired open-loop transfer function be represented in the form $H_{opt}(p)$ .Such a representation makes it possible to establish the necessary relationship between the system parameters and its performance indicators. Furthermore, the use of a desired transfer function provides a direct basis for applying structural and parametric synthesis methods to determine the characteristics of a controller capable of ensuring compliance with the specified control requirements.

Since the system operates according to the principle of negative feedback, its dynamic properties are determined not only by the parameters of the open-loop path but also by the interaction between the forward and feedback channels. The presence of negative feedback reduces the control error, improves the accuracy of maintaining the desired controlled variable, and enhances the robustness of the system against external disturbances and parametric variations of the controlled plant.

Based on the classical relationships of automatic control theory, using the open-loop transfer function and taking into account unity negative feedback, the transfer function of the closed-loop system $H_C(p)$ can be obtained. This transfer function determines the system response to reference inputs and makes it possible to evaluate the main performance indicators, such as settling time, overshoot, steady-state error, and dynamic error.

Thus, the subsequent analysis of the synthesized system will be carried out on the basis of the closed-loop transfer function derived from the desired open-loop transfer function. Therefore, representing the desired open-loop transfer function in the form $H_{opt}(p)$ and taking into account unity negative feedback, we obtain the following closed-loop transfer function $H_C(p)$:

$$\begin{aligned} H_{opt}(p)&=\frac{b_{opt}p+1}{a_{opt}p^{1+\mu}}\frac{1}{T_\nu p+1}.\\ H_C(p)&=\frac{a_{opt}p^{1+\mu}T_\nu p+a_{opt}p^{1+\mu}}{a_{opt}p^{1+\mu}T_\nu p+a_{opt}p^{1+\mu}+b_{opt}p+1}. \end{aligned} \qquad (2)$$

Such a loop possesses an astatism order of $1+\mu$, which is one of its most important features and determines the system’s ability to effectively compensate for both steady-state and dynamic control errors. The presence of a fractional astatism order makes it possible to combine the advantages of classical astatic systems with the additional capabilities provided by the mathematical apparatus of fractional calculus. As a result, the system acquires improved properties in terms of tracking reference inputs and rejecting external disturbances.

It is well known that first-order astatic systems ensure the complete elimination of steady-state error when responding to constant reference signals. However, when the reference signal varies with time, particularly in the presence of linearly increasing or other time-varying inputs, dynamic errors arise within the system. The application of a fractional astatism order makes it possible to significantly reduce the magnitude of such errors without substantially increasing the complexity of the system structure or requiring excessively high controller gains.

For a loop with an astatism order of $1+\mu$, not only is the steady-state error completely eliminated, but the dynamic components of the control error are also effectively reduced. In particular, the velocity error that occurs under a linearly varying reference signal is no longer a constant quantity, as in conventional systems, but gradually decreases with time and tends toward zero. This property is especially important for automatic control systems of technological processes, where high accuracy in maintaining the controlled variable must be ensured under continuously changing operating conditions.

Furthermore, increasing the astatism order has a positive effect on transient response quality and the system’s ability to compensate for external disturbances. This is particularly relevant for cutting-power stabilization systems, where the load may vary rapidly due to fluctuations in machining allowance, non-uniformity of the workpiece material, or changes in the condition of the cutting tool. Under such conditions, a system with a fractional astatism order provides more accurate tracking of reference inputs and more effective disturbance rejection compared with conventional control structures.

Therefore, the use of a loop with an astatism order of $1+\mu$ creates the conditions necessary for simultaneously achieving high control accuracy, minimal steady-state and dynamic errors, and improved transient response characteristics. These properties justify its application in the synthesis of advanced automatic cutting-power stabilization systems for milling machines.

The optimal coefficients $a_{opt}$ and $b_{opt}$ are determined to ensure that the transient process corresponding to a unit-step input satisfies the following optimization requirements:

- minimization of overshoot;
- maximization of system response speed.

These requirements form the basis for selecting the parameters of the desired transfer function and synthesizing the corresponding control system.

Let us select an analytical formulation of the transient-response performance criterion that will be used in the parametric optimization of the closed-loop control system. The final result of the synthesis procedure largely depends on the proper choice of this criterion, since it determines the direction of the search for optimal system parameters and establishes the requirements for the dynamic behavior of the control system. For cutting-power stabilization systems, it is particularly important to ensure not only a high response speed but also to limit the maximum deviation of the controlled variable from its prescribed value. This requirement is обусловved by the fact that even short-term excesses of the permissible power level may result in deterioration of machining quality, overloading of the electric drive, and accelerated wear of the cutting tool.

The classical integral mean-square error criterion can formally be represented as the sum of the products of the instantaneous error values and their corresponding weights, which numerically coincide with the error values themselves. Under such conditions, the largest contribution to the objective function is produced by the initial stages of the transient process, where the deviation of the controlled variable from the desired value is maximal. Consequently, during optimization the system tends to reduce the initial error as rapidly as possible, resulting in transient responses characterized by the highest achievable response speed.

From a mathematical perspective, such an approach is entirely reasonable because minimization of the integral quadratic criterion ensures a reduction of the overall error level throughout the entire transient process. However, for practical technological systems this criterion does not always reflect the actual control requirements. As a result of optimization, a transient response is often obtained that may be described as being “pressed” toward the ordinate axis. Such a response exhibits very high speed, but this advantage is achieved at the expense of considerable short-term overshoot and increased dynamic loading.

A particularly significant drawback of the classical mean-square criterion is its low sensitivity to short-duration excursions beyond the specified level. Even when the overshoot reaches relatively large values, its contribution to the overall criterion may remain comparatively small if its duration is short. Consequently, the optimization algorithm may regard such a response as acceptable and even preferable to alternatives characterized by slightly lower response speed but significantly better accuracy and robustness.

For automatic cutting-power stabilization systems, such an approach is unacceptable. In machining processes, even a short-term excess of the permissible power level may cause feed-drive overload, increased temperature in the cutting zone, elevated mechanical stresses within the technological system, and deterioration of the machined surface quality. Moreover, excessive peak loads adversely affect tool life and may lead to premature tool failure. Therefore, minimizing only the integral mean-square error does not guarantee the achievement of the most suitable dynamic performance for technological applications.

Consequently, the use of the classical mean-square performance criterion alone does not provide the necessary compromise between response speed and limitation of maximum deviations of the controlled variable. This creates the need for the development of a specialized transient-response quality criterion that simultaneously accounts for the magnitude of the control error, the duration of the transient process, and the permissible overshoot level. Such an approach makes it possible to obtain dynamic characteristics that are most appropriate for automatic cutting-power stabilization systems of milling machines and ensures compliance with the stringent requirements imposed by modern machining processes.

Let us formulate a transient-response optimization criterion for systems that are required to provide high response speed (characterized by the selected value of $T_\nu$ while maintaining the response within a prescribed tolerance corridor of width $\pm\delta$ around the desired value (Fig. 1).

For this purpose, the control errors at each calculation step are accumulated until the transient response first enters the admissible corridor $\left(|Y_i - 1| < \delta\right)$, in which case their weighting factor is equal to unity. Subsequently, the objective functional (F) is increased by the fourth power of the error normalized with respect to δ. Consequently, within the admissible corridor the weighting factor becomes less than one, whereas outside the corridor it exceeds one and increases according to a cubic-parabolic law:

$$F = \sum_{i=1}^{N} \begin{cases} (1 - Y_i)\ \forall\, t < t_1 \\ \left(\dfrac{1 - Y_i}{\delta}\right)^4 \forall\, t > t_1 \end{cases} \rightarrow \min, \qquad (3)$$

where $t_1$ is the time of the first entry into the admissible corridor.

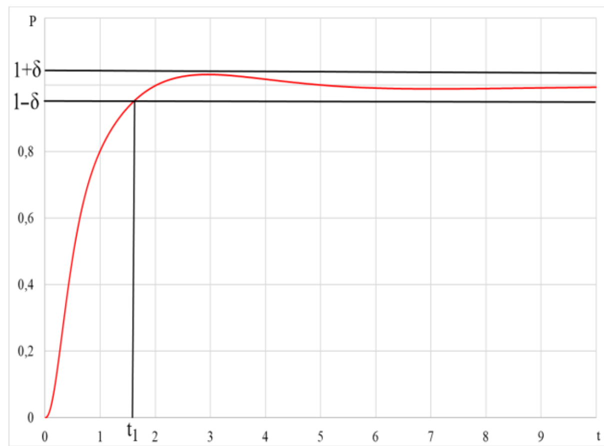


Figure 1 – Criterion for Evaluating Transient Response Quality

The proposed performance criterion eliminates one of the major drawbacks of conventional integral quadratic criteria, namely the tendency of the optimization procedure to compress the transient response toward the origin of the time axis. As a result, the use of classical criteria often produces transient responses characterized by extremely high response speed accompanied by significant short-term overshoots or undershoots of the controlled variable. For many technical systems, such deviations may be considered acceptable if their duration is sufficiently short. However, for automatic control systems of technological processes, such behavior is undesirable.

The proposed criterion is aimed at generating transient responses that provide not only high response speed but also compliance with prescribed limitations on the allowable deviations of the controlled variable. By introducing a special penalty mechanism for errors occurring outside the specified tolerance corridor, the optimization procedure receives additional information regarding the undesirability of significant local extrema. As a consequence, the optimal transient response becomes smoother and is characterized by reduced overshoot and undershoot levels.

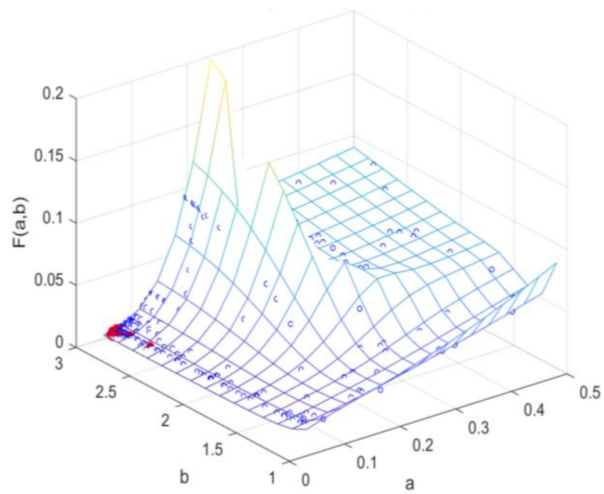
An important advantage of the proposed approach is its ability to establish a compromise between the speed of reaching the steady-state operating condition and the overall quality of the transient response. While conventional criteria are primarily focused on minimizing the integral error, the proposed performance index additionally accounts for the permissible limits of deviation of the controlled variable. As a result, the optimization process is directed not only toward reducing the duration of the transient response but also toward ensuring its technological acceptability. This feature is particularly important for cutting-power stabilization systems, where even a short-term excess of the permissible load level may lead to deterioration of machining quality or equipment overload.

The search for the parameters of the optimal transfer function was carried out using genetic algorithms. The choice of this evolutionary optimization approach was motivated by the complexity of the optimization problem and the specific characteristics of the mathematical model. In particular, for systems with fractional-order integration, the shape of the response surface strongly depends on the selected astatism order and other model parameters. Under such conditions, the objective function may exhibit a complex multimodal structure containing numerous local minima and maxima.

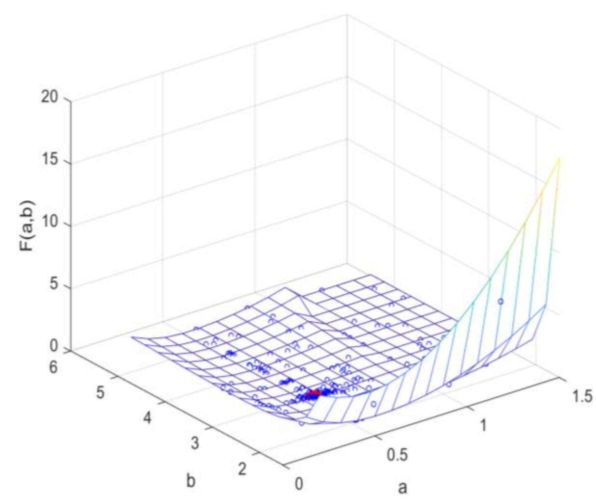
The application of conventional gradient-based optimization methods to such problems is frequently associated with the risk of convergence to a local minimum. In this case, the obtained solution may be only locally optimal and may differ substantially from the global optimum. This problem becomes particularly significant in the synthesis of fractional-order systems, where even a small variation in the integration order may result in a considerable transformation of the objective-function surface.

Unlike gradient-based methods, genetic optimization algorithms perform a simultaneous search across multiple regions of the parameter space and employ the mechanisms of natural selection, crossover, and mutation to generate new candidate solutions. As a result, the probability of locating the global minimum of the performance functional is significantly increased, even in the presence of highly complex response surfaces. Furthermore, genetic algorithms do not require the computation of derivatives of the objective function, which constitutes an important advantage when dealing with nonlinear models and fractional-order operators.

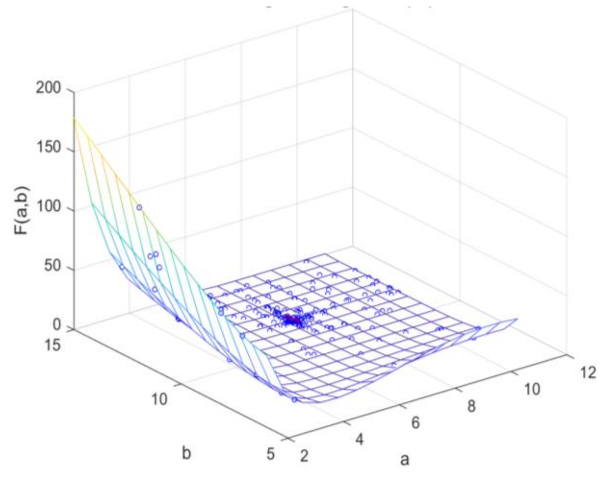
Therefore, the combination of the proposed transient-response performance criterion with genetic optimization algorithms makes it possible to determine controller parameters that simultaneously provide high response speed, limited overshoot, and high accuracy of cutting-power stabilization. Such a combination of methods creates the prerequisites for the synthesis of efficient automatic control systems capable of operating under variable technological loads while ensuring stable quality of the machining process (Fig. 2).



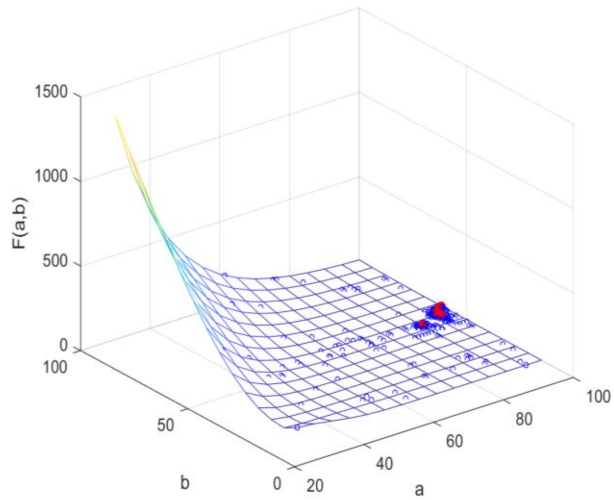

a)



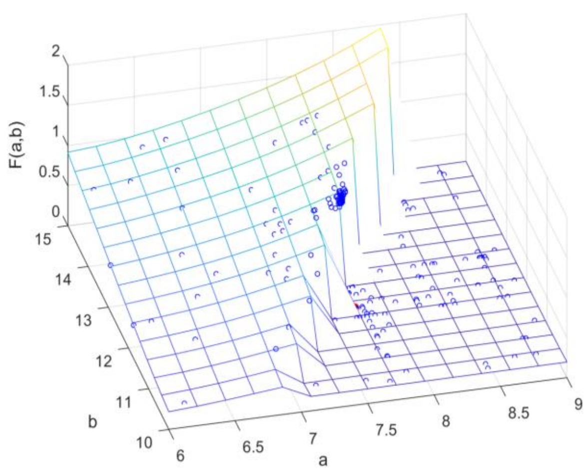

b)



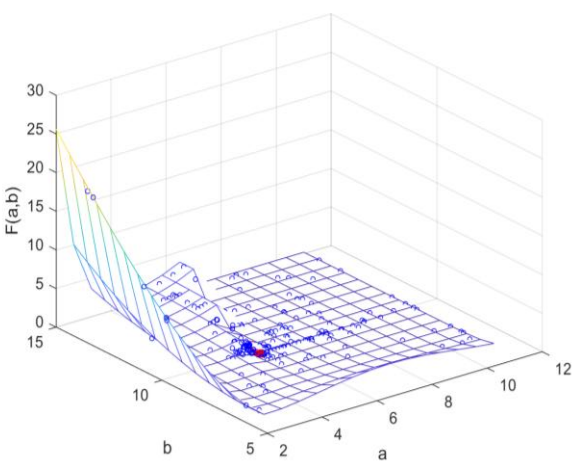

c)




f)




d)




e)

Figure 2 – Response surfaces corresponding to optimization criterion (3) for different values of the fractional integration order: (a) μ=0.1, (b) μ=0.2, (c) μ=0.5, (d) μ=0.6, (e) μ=0.7, and (f) μ=0.9

The genetic algorithm in the MATLAB environment was configured to ensure a reliable search for the global minimum of the performance functional while minimizing the risk of premature convergence to local extrema. The selection of algorithm parameters was performed taking into account the specific characteristics of the optimization problem associated with fractional-order systems, which are characterized by complex multimodal response surfaces and the presence of numerous local minima. The primary objective of the tuning procedure was to achieve an optimal balance between the convergence speed of the algorithm and the quality of the obtained solution.

The initial population was generated randomly within the permissible ranges of variation of the optimized parameters. Such an approach ensured a uniform coverage of the search space at the initial stage of optimization and created the necessary conditions for exploring different regions of the objective-function surface. The population size was selected to be sufficiently large to preserve genetic diversity and prevent the premature loss of promising search directions.

The quality of each individual was evaluated using the proposed transient-response optimization criterion, which simultaneously accounted for system response speed and the magnitude of deviations of the controlled variable from the admissible tolerance corridor. The value of the performance functional was calculated for each candidate parameter set by simulating the dynamic behavior of the closed-loop control system and subsequently analyzing the resulting transient response.

The parent-selection procedure was based on the principle of favoring more fit individuals. Candidate solutions associated with lower values of the objective function were assigned a higher probability of participating in the generation of offspring. Such a mechanism allowed the search process to gradually concentrate on the most promising regions of the parameter space while maintaining sufficient population diversity.

New candidate solutions were generated using crossover and mutation operators. The crossover operator combined the parameters of two parent individuals to produce offspring inheriting advantageous properties from both solutions. The mutation operator introduced random modifications to selected parameters, thereby preventing the optimization process from becoming trapped in local extrema and maintaining the diversity of the population throughout the search procedure.

Particular attention was devoted to the selection of crossover and mutation probabilities. Excessively high mutation rates could lead to the loss of valuable information accumulated during previous generations, whereas excessively low mutation rates could result in premature convergence. Therefore, the parameters of the genetic operators were selected so as to provide an effective compromise between the exploration of new regions of the search space and the exploitation of already identified promising solutions.

The termination criterion of the algorithm was defined either by reaching a prescribed number of generations or by the absence of a significant improvement in the objective-function value over a specified number of iterations. Upon completion of the optimization procedure, the individual with the minimum value of the performance functional among all evaluated solutions was selected as the optimal one. The resulting parameters were then used to construct the desired transfer function and to synthesize the corresponding fractional-integral controller.

The application of the genetic algorithm in MATLAB made it possible to efficiently explore a multidimensional parameter space, identify globally optimal values of the system coefficients, and obtain transient responses characterized by high response speed, limited overshoot, and minimal dynamic errors. These results confirm the effectiveness and suitability of evolutionary optimization methods for the synthesis of fractional-order automatic control systems.

- CreationFcn: @gacreationuniform – the initial population generation function. The gacreationuniform option was selected so that the initial individuals are distributed uniformly and randomly throughout the entire search space bounded by lb (lower bounds) and ub (upper bounds). This approach ensures broad coverage of the parameter space and increases the probability of identifying promising regions during the early stages of optimization.
- CrossoverFcn: @crossoverscattered – the crossover function. The crossoverscattered operator selects two parent individuals and randomly chooses genes (decision variables) from each parent to create an offspring. This mechanism provides a high degree of variability in the generated population and promotes effective exploration of the search space.
- SelectionFcn: @selectionstochunif – the parent selection function. The selectionstochunif option implements stochastic uniform selection, in which the probability of selecting an individual is proportional to its fitness while retaining a degree of randomness. This approach provides a balance between the principle of “survival of the fittest” and the preservation of population diversity.
- MutationFcn: @mutationadaptfeasible – the mutation function. The mutationadaptfeasible operator adaptively modifies the genes of individuals while ensuring that all parameters remain within the specified admissible bounds (lb, ub). This strategy helps prevent premature convergence, maintains genetic diversity, and improves the algorithm’s ability to escape local minima and continue searching for the global optimum.

The obtained optimal tuning parameters are summarized in Table 1. Figure 3 presents the three-dimensional relationship $F_{opt} = f\left(a_{pt},\ b_{opt}\right)$ while Figure 4 shows the relationship $b_{opt} = f\left(a_{opt}\right)$.

Table 1. Normalized Parameters of the Optimal Controller Tuning

| $1+\mu$ | $a_{opt}^{norm}$ | $b_{opt}^{norm}$ | F |
|---|---|---|---|
| 1.10 | 0.05652 | 2.94932 | 0.003371 |
| 1.20 | 0.23151 | 2.22772 | 0.017871 |
| 1.30 | 0.55071 | 2.88831 | 0.039412 |
| 1.40 | 1.11051 | 4.13712 | 0.082162 |
| 1.50 | 2.25753 | 6.09612 | 0.133783 |
| 1.55 | 5.25321 | 10.06211 | 0.128072 |
| 1.60 | 7.47841 | 11.94541 | 0.141791 |

| | | | |
|---|---|---|---|
| 1.65 | 7.23282 | 11.61062 | 0.142022 |
| 1.70 | 11.06963 | 14.88031 | 0.134943 |
| 1.80 | 29.19471 | 27.56051 | 0.147751 |
| 1.90 | 99.84562 | 61.42632 | 0.205681 |

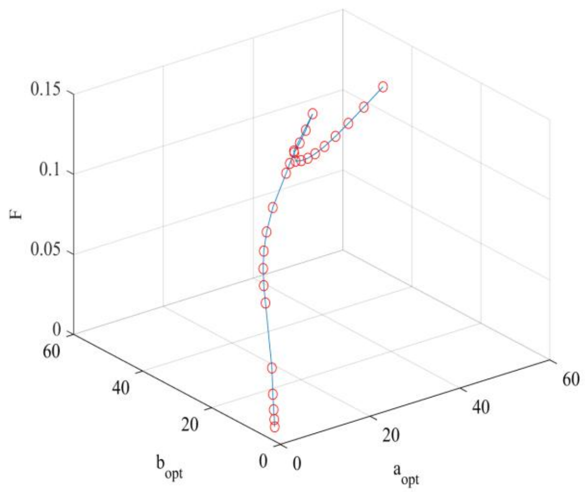



Figure 3 – Three-dimensional relationship
$F_{opt} = f\left(a_{pt},\ b_{opt}\right)$

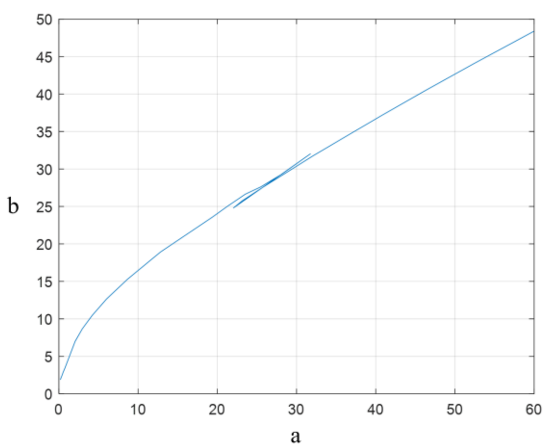



Figure 4 – Relationship $b_{opt} = f\left(a_{opt}\right)$

Figures 3 and 4 clearly show the presence of a loop in the region $1.651 < 1+\mu < 1.751$ which complicates the derivation of analytical functions $a_{opt} = f(\mu),\ b_{opt} = f(\mu)$ but does not affect the achievement of high-quality transient-response characteristics. Figure 5 presents families of transient-response functions in the normalized time scale $\dfrac{t}{T_v}$ obtained on the basis of the proposed optimization criterion. It can be seen that in all cases the overshoot is lower than that obtained using the classical modulus (technical) optimum tuning method (4.352 %, corresponding to $y_{\max} = 1.0435$ while providing significantly higher response speed. For all values of $\mu$, the first settling occurs before $4.7T_v$ and the response speed increases with increasing astatism order $\mu$.

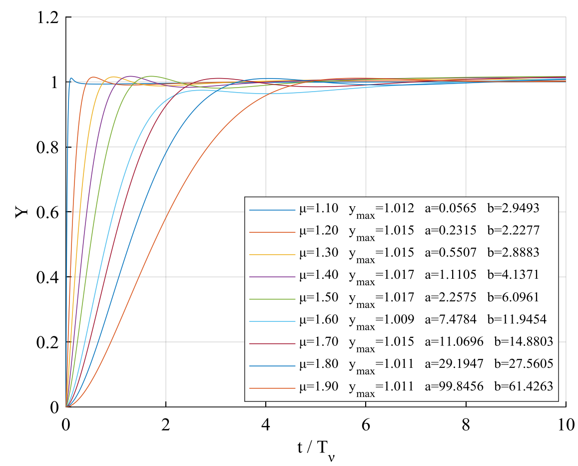



Figure 5 – Transient responses corresponding to the optimal values of $\mu$, $a$, $b$, and ($\delta$ = 0.025) according to criterion (3).

These functions are normalized with respect to both the horizontal and vertical axes and therefore must be converted into absolute units for a particular technological process. The most challenging task is the time scaling procedure. During the calculation of the parameters (a) and (b), the value $T_v = 1$ was assumed. Consequently, the Laplace operator (p) corresponds to real time according to the relation:

$$p \leftrightarrow T_v \frac{d}{dt}. \tag{4}$$

Then, from transfer function (2), we obtain:

$$H_{opt}^{norm}(p) = \frac{b_{opt}^{norm} p + 1}{a_{opt}^{norm} p^{1+\mu}} \frac{1}{p+1} \Rightarrow$$
$$\Rightarrow H_{opt}(p) = \frac{\left(b_{opt}^{norm} T_v\right) p + 1}{\left(a_{opt}^{norm} T_v^{1+\mu}\right) p^{1+\mu}} \frac{1}{T_v p + 1} \tag{5}$$

Thus, in the actual system

$$a_{opt} = a_{opt}^{norm} T_{\nu}^{1+\mu};$$
$$b_{opt} = b_{opt}^{norm} T_{\nu}. \qquad (6)$$

This approach makes it possible to obtain optimal controller settings for a wide class of automatic control systems in which stringent requirements are imposed on transient-response quality, including high response speed, control accuracy, and robustness against external disturbances. The proposed methodology enables the determination of controller parameters in such a way as to ensure minimal steady-state and dynamic errors while maintaining an acceptable overshoot level and a high rate of response to reference inputs.

A particularly important advantage of the developed approach is that the obtained relationships between the parameters of the optimal transfer function and the astatism order can be used directly during the control-system design stage without the need to repeatedly perform a computationally intensive optimization procedure for each new application. This significantly reduces the time required for system synthesis and simplifies the selection of rational controller parameters.

The proposed method also makes it possible to account for the specific requirements of a particular technological process through the appropriate selection of the admissible deviation corridor and the required response speed. As a result, the controller parameters can be adapted to the operating conditions of the actual controlled plant, making it possible to achieve an optimal compromise between system responsiveness and the accuracy of maintaining the controlled variable at its prescribed value.

The practical implementation of the proposed approach was carried out for the automatic cutting-power control system of a milling machine. For this system, optimal parameters of fractional-integral controllers were determined, providing effective disturbance rejection, reduced dynamic deviations of cutting power, and improved transient-response characteristics compared with conventional tuning methods. The obtained results confirmed the effectiveness of the proposed methodology and demonstrated its suitability for application in modern automatic control systems for technological processes.

Thus, the developed approach establishes both a theoretical and practical foundation for the synthesis of high-performance fractional-order control systems. It can be applied to the design of automated stabilization systems for technological parameters, particularly cutting power, in various types of metalworking equipment. Furthermore, the proposed methodology may serve as a basis for the development of advanced control systems capable of maintaining high performance and stability under variable operating conditions, thereby improving machining quality, increasing equipment productivity, and enhancing the overall efficiency of manufacturing processes.

**Synthesis of Fractional-Order Controllers for a Cutting Power Stabilization System**.

One of the key stages in the development of high-performance automatic control systems is the design of controller synthesis methods capable of ensuring the required control quality under conditions of plant-parameter uncertainty and external disturbances. This task is particularly important for cutting-power control systems, where the parameters of the technological process may vary over a wide range depending on the properties of the workpiece material, cutting conditions, tool wear, and other influencing factors. Under such conditions, conventional integer-order controllers do not always provide the required levels of accuracy and response speed, which justifies the application of fractional-integral controllers.

A significant advantage of fractional-integral controllers is the presence of additional tuning parameters associated with fractional-order integration and differentiation. These additional degrees of freedom make it possible to shape the dynamic characteristics of the system more flexibly, provide the required stability margins, and achieve a better compromise between response speed, control accuracy, and overshoot. As a result, fractional-order systems are finding increasingly widespread application in automatic control problems involving complex technological objects.

An important stage in the synthesis of fractional-integral controllers is the selection of an appropriate mathematical model of the controlled plant. It is well known that many real technological and electromechanical systems exhibit complex inertial properties that cannot always be adequately represented by conventional integer-order models. In such cases, fractional-order models provide a more accurate description of the underlying physical processes by taking into account memory effects, distributed parameters, and the complex dynamic interactions among the individual components of the system.

In many cases, the controlled plant can be described by the following fractional-order transfer function [6]. Such a model represents a generalization of conventional transfer functions and allows the fractional integral-differential properties of the plant to be taken into account. The use of this representation provides a more accurate mathematical description of the system and creates the basis for designing controllers whose parameters are consistent with the actual dynamic characteristics of the controlled object.

A fractional-order transfer function may include both fractional powers of the Laplace operator and coefficients characterizing the inertial and gain properties of the plant. The values of these parameters are determined from experimental data or through the identification of a mathematical model. Subsequently, these parameters are used during the parametric synthesis of the fractional-integral controller and for determining its optimal tuning settings.

Therefore, the use of fractional-order models provides the necessary theoretical foundation for the synthesis of advanced cutting-power control systems by improving the accuracy of the plant representation and enhancing the quality of automatic regulation. Consequently, in the following analysis, it will be assumed that the controlled plant is described by the following fractional-order transfer function [6]:

$$H_O^{\,s}(p)=\frac{k}{a_2 p^{1+\mu(s)}+a_1 p^{\mu(s)}+1}\frac{1}{T_\nu p+1}, \quad (7)$$

where $k,\ a_2,\ a_1,\ \mu$ denote the parameters of the controlled object.

To synthesize a controller that will be connected in series with the controlled plant and provide the required dynamic characteristics of the closed-loop system, it is necessary to determine its transfer function in such a way that the combined operation of the controller and the plant ensures the realization of the desired optimal open-loop transfer function. In essence, the synthesis problem consists in finding a controller structure and parameter set capable of transforming the dynamic properties of the actual plant into the desired system characteristics specified during the previous design stage.

It is well known that the behavior of a closed-loop control system is directly determined by the product of the transfer functions of the controller and the controlled plant. For this reason, the controller synthesis procedure can be reduced to solving an equation that establishes the relationship between the desired open-loop transfer function and the mathematical model of the plant. By solving this equation, the analytical form of the controller transfer function can be obtained, ensuring that the specified control-performance requirements are achieved.

A distinctive feature of the present problem is that the controlled plant is described by a fractional-order model. Consequently, the synthesized controller must also contain fractional-order integral and/or differential elements whose parameters are coordinated with the fractional characteristics of the plant. Such an approach preserves the advantages of the fractional-order system representation and enables the full utilization of the additional degrees of freedom provided by fractional calculus.

From a mathematical point of view, the synthesis problem is reduced to determining the unknown controller transfer function by equating the product of the controller and plant transfer functions to the desired open-loop transfer function. The resulting relationship serves as the fundamental equation for the subsequent parametric synthesis procedure and for determining the structure of the fractional-integral controller.

Therefore, in order to construct a controller connected in series with the controlled plant and capable of providing the required dynamic characteristics of the system, it is necessary to find the solution of the following equation:

$$H_{opt}(p)=H_R(p)H_O^{\,s}(p)\Rightarrow \quad (8)$$

$$\frac{\left(b_{opt}^{norm}T_\nu\right)p+1}{\left(a_{opt}^{norm}T_\nu^{\,1+\mu}\right)p^{1+\mu}}\frac{1}{T_\nu p+1}=$$
$$=H_R(p)\frac{b(s)}{a_2(s)p^{1+\mu(s)}+a_1(s)p^{\mu(s)}+1}\frac{1}{T_\nu p+1}, \quad (9)$$
$$\Rightarrow$$

$$H_R(p)=\frac{\left(b_{opt}^{norm}T_\nu\right)p+1}{\left(a_{opt}^{norm}T_\nu^{\,1+\mu}\right)p^{1+\mu}}\times$$
$$\times\frac{a_2(s)p^{1+\mu(s)}+a_1(s)p^{\mu(s)}+1}{b(s)},\ \Rightarrow \quad (10)$$

$$H_R(p)=\frac{\left(\left(b_{opt}^{norm}T_\nu\right)p+1\right)}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)}\times$$
$$\times\left(\frac{a_2(s)p^{1+\mu(s)}}{p^{1+\mu}}+\frac{a_1(s)p^{\mu(s)}}{p^{1+\mu}}+\frac{1}{p^{1+\mu}}\right). \quad (11)$$

Equation (11) makes it possible to determine both the structure and the parameters of the controller. Based on this relationship, various controller configurations can be synthesized depending on the desired dynamic characteristics of the closed-loop system, the properties of the controlled plant, and the specified performance requirements.

Let us consider several possible controller structures and evaluate their suitability for the cutting-power control system. The choice of controller structure is a crucial stage of the synthesis procedure because it directly affects the dynamic and static performance of the closed-loop system. Different controller configurations provide different levels of response speed, control accuracy, disturbance rejection, and overshoot suppression.

Fractional-order control systems offer greater flexibility than conventional integer-order controllers due to the additional parameters associated with fractional integration and differentiation. By properly selecting these parameters, it becomes possible to achieve an improved compromise between speed of response, stability, and control accuracy.

For the cutting-power stabilization system, the controller must maintain the desired power level under varying load conditions while ensuring rapid disturbance compensation and limited overshoot. Therefore, several controller structures are analyzed to identify the most effective solution.

The synthesis procedure is based on matching the desired open-loop transfer function with the product of the controller and plant transfer functions. Depending on the selected fractional-order parameter, different controller structures can be obtained, each characterized by specific dynamic properties.

As a first case, let us consider the situation in which the desired fractional-order parameter is defined as $\mu=\mu(s)$ The corresponding controller structure can then be derived from the synthesis equation presented above.

Desired parameter $\mu=\mu(s)$:

$$H_R(p)=\frac{\left(\left(b_{opt}^{norm}T_\nu\right)p+1\right)}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)}\times$$
$$\times\left(a_2(s)+a_1(s)\frac{1}{p}+\frac{1}{p^{1+\mu}}\right)\Rightarrow$$
$$H_R(p)=\frac{1}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)}\times$$
$$\times\begin{pmatrix}\left(b_{opt}^{norm}T_\nu\right)a_2(s)p+\\ +a_2(s)+\left(b_{opt}^{norm}T_\nu\right)a_1(s)+\\ +a_1(s)\frac{1}{p}+\\ +\frac{1}{p^{\mu}}\left(\left(b_{opt}^{norm}T_\nu\right)+\frac{1}{p}\right)\end{pmatrix}\Rightarrow$$
$$H_R(p)=\left(k_d p+k_p+\frac{k_i}{p}\right)+$$
$$+I^{\mu}\left(k_{p\mu}+\frac{k_{i\mu}}{p}\right). \quad (12)$$

The resulting controller has the structure $(PID)+I^{\mu}(PI)$, where the parameters.

$$k_d=\frac{\left(b_{opt}^{norm}T_\nu\right)a_2(s)}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)},$$
$$k_p=\frac{a_2(s)+\left(b_{opt}^{norm}T_\nu\right)a_1(s)}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)},$$
$$k_i=\frac{a_1(s)}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)},$$
$$k_{p\mu}=\frac{\left(b_{opt}^{norm}T_\nu\right)}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)},$$
$$k_{i\mu}=\frac{1}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)}.$$

It is important that the operation of calculating the fractional integral (the most complex one) is performed only once.

Desired parameter $\mu\neq\mu(s)$:

$$H_R(p)=\frac{\left(\left(b_{opt}^{norm}T_\nu\right)p+1\right)}{\left(a_{opt}^{norm}T_\nu^{1+\mu}\right)b(s)}\times \left(\frac{1}{p^{\mu-\mu(s)}}\left(a_2(s)+\frac{a_1(s)}{p}\right)+\frac{1}{p^{1+\mu}}\right). \quad (13)$$

In this case, a controller with the structure $(PD)\left(I^{\mu-\mu(s)}(PI)+I^{1+\mu}\right)$ has been obtained. It includes two fractional-integral elements of small positive or negative order $\mu-\mu(s)$ and of order $1+\mu$.

The obtained controllers provide the closed-loop system response shown in Figs. 3 and 5, respectively.

Let us implement the system model and investigate the start-up process with a linearly increasing reference signal, as well as the response to an instantaneous increase and decrease in the depth of cut, which within 0.05 s leads to a 50% change in cutting power. The coefficients are normalized for $T_\nu = 0.01\ s$, which is a typical standard time constant of electric drives.

Figure 6 shows the transient response for different selected integration orders $1+\mu$. It can be seen that the dynamic errors for an order of 1.5 are smaller than those for an order of 1.8. However, taking into account the identification results and the need to simplify the controller structure, tuning with an order in the range of 1.7…1.8 should be considered the most appropriate.

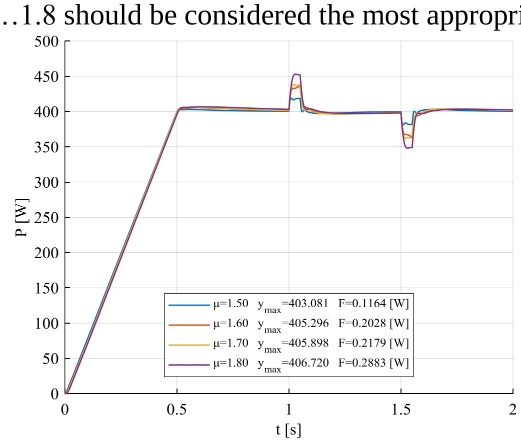



Figure 6 – Transient characteristics of cutting power during start-up and under sudden load increase and decrease

Figure 7 shows enlarged sections of the load increase and load decrease transients, which confirm the effectiveness of the proposed control system. The control action, implemented through the corresponding adjustment of the feed rate, significantly reduces the amplitude of the cutting-power disturbance from 200 W to 19 W at μ=1.5 and to 39 W at the recommended value of μ=1.7. The settling time of the transient response does not exceed 0.02 s.

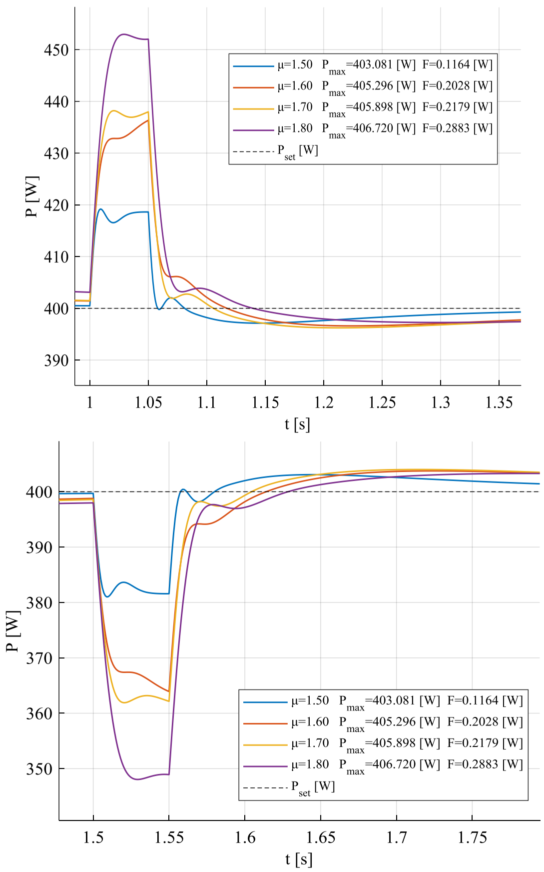



Figure 7 – Enlarged Sections of the Transient Responses to Load Application and Load Removal

Figure 8 illustrates one of the most important properties of a system with a fractional astatism order greater than unity, namely the gradual reduction of the velocity error under a linearly varying reference signal. Unlike classical first-order astatic systems, in which the velocity error approaches a constant value, systems with a fractional astatism order of (1+μ) exhibit a continuously decreasing velocity error that tends toward zero as time increases. This distinctive feature is a consequence of the additional integrating properties introduced by the use of fractional-order operators.

The obtained result demonstrates the ability of the system to track time-varying reference inputs more accurately and to compensate dynamic errors more effectively than conventional control structures. This property is particularly important for automatic cutting-power control systems, where operating conditions continuously change due to technological factors such as variations in workpiece properties, cutting depth, and tool condition. Owing to the gradual reduction of the velocity error, the system is capable of maintaining the desired cutting-power level with higher accuracy, thereby improving machining quality and reducing the influence of external disturbances on system performance.

Furthermore, the tendency of the velocity error to approach zero indicates an improvement in the dynamic accuracy of the control system without requiring a substantial increase in controller gain. As a result, systems with a fractional astatism order greater than one provide an effective means of enhancing control performance while preserving favorable transient-response characteristics and maintaining a sufficient stability margin. These advantages make fractional-order control structures particularly attractive for high-precision technological applications in which both dynamic performance and regulation accuracy are of primary importance.

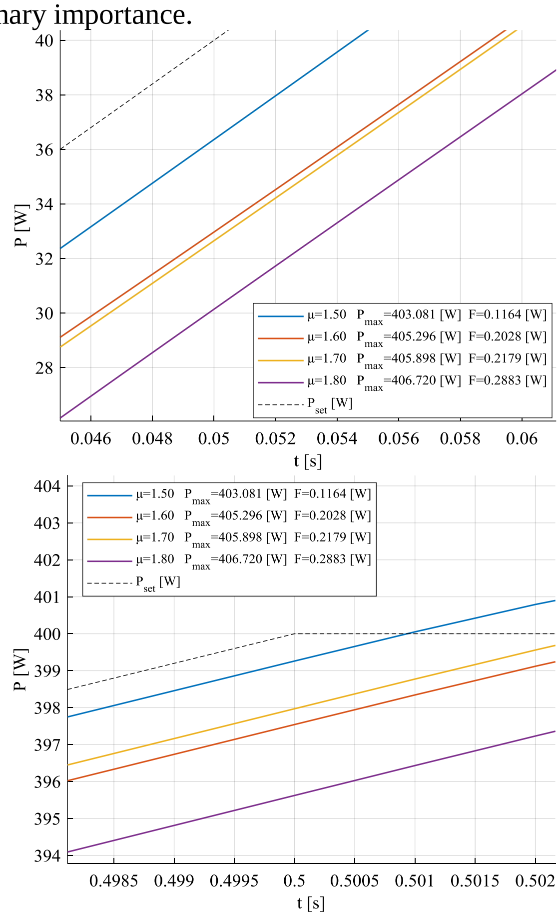



Figure 8 – Enlarged Fragments of the Initial and Final Stages of the Start-Up Process with a Linearly Increasing Reference Signal

At the beginning of the transient process, the control errors for all considered tuning variants are approximately two to four times greater than those observed at the final stage of the start-up process. This behavior is typical of systems operating under a linearly increasing reference signal, since the controlled variable initially experiences the largest dynamic mismatch caused by the limited bandwidth of the control loop and the finite response rate of the system. As the transient process progresses, the influence of the initial mismatch gradually decreases, enabling the control system to follow the reference signal with significantly higher accuracy.

The analysis of the obtained results indicates that the system with an astatism order of 1.5 exhibits the smallest dynamic error over the entire operating range and therefore provides the highest tracking accuracy. The reduced velocity error and improved transient-response characteristics of this configuration are direct consequences of its dynamic properties and the selected controller parameters. From a purely theoretical control perspective, this system can be regarded as the most accurate among the investigated alternatives.

However, it should be emphasized that the objective of the considered control system is not the stabilization of speed or position, where maximum tracking accuracy is usually the primary design criterion. In the present study, the controlled variable is the cutting power of a milling process. For such technological systems, the requirements imposed on control accuracy differ considerably from those associated with motion-control applications. Small residual dynamic errors generally have only a minor influence on the overall quality of the machining process, provided that the cutting power remains within the permissible operating range.

Therefore, when selecting the optimal astatism order, not only the achievable control accuracy but also the practical complexity of controller implementation must be taken into account. An increase in the astatism order generally leads to a more complicated controller structure, a greater number of fractional-order elements, and increased computational effort during real-time operation. Consequently, the relatively small improvement in accuracy achieved with an astatism order of 1.5 may not justify the additional implementation complexity.

For this reason, the final controller structure should be selected on the basis of a compromise between regulation accuracy and implementation simplicity. Since the differences in cutting-power stabilization accuracy among the investigated configurations are relatively small, the complexity of the controller becomes the dominant selection criterion. From this standpoint, astatism-order values within the range of 1.7–1.8 should be considered the most rational choice, as they provide sufficiently high control quality while maintaining an acceptable level of structural and computational complexity.

## IV. RESULTS AND DISCUSSION

The proposed methodology for the synthesis of closed-loop cutting-power stabilization systems was evaluated through numerical optimization and simulation studies. The optimization procedure was based on the transient-performance criterion formulated in Eq. (3), which simultaneously accounts for response speed and compliance with a predefined corridor of permissible deviations. Unlike classical integral quadratic criteria, the proposed functional penalizes deviations outside the admissible range and therefore promotes the formation of transient responses with limited overshoot while preserving high dynamic performance.

The optimization of the desired transfer-function parameters was performed using a genetic algorithm implemented in MATLAB. The obtained response surfaces demonstrated a pronounced dependence of the objective function on the fractional astatism order. As shown in Fig. 2, the shape of the optimization landscape changes significantly with increasing values of the fractional parameter μ, confirming the presence of multiple local minima and justifying the use of evolutionary optimization methods instead of gradient-based approaches. The genetic algorithm successfully identified globally optimal parameter combinations for all investigated values of the fractional integration order.

The normalized optimal parameters obtained during the optimization process are summarized in Table 1. Analysis of the results indicates that both coefficients ($a_{opt}$) and ($b_{opt}$) increase nonlinearly with increasing astatism order. For values of (1+μ) below approximately 1.5, the growth of the coefficients is relatively moderate, whereas for larger astatism orders the parameters increase rapidly. Such behavior reflects the increasing dynamic complexity of the desired closed-loop system and the necessity of stronger compensating actions to achieve the required transient performance.

The relationships between the optimized parameters are illustrated in Figs. 3 and 4. A characteristic loop can be observed in the region ($1.65 < 1+\mu < 1.75$), indicating the presence of multiple parameter combinations that provide nearly identical values of the objective function. Although this phenomenon complicates the derivation of analytical approximations ($a_{opt}=f(\mu)$) and ($b_{opt}=f(\mu)$), it does not noticeably affect the quality of the resulting transient responses. Consequently, the optimization procedure remains robust within this parameter range.

The synthesized fractional-integral controllers were subsequently implemented in the cutting-power stabilization system and evaluated under various operating conditions. Figure 5 presents the family of normalized transient responses corresponding to the optimal parameter combinations. It can be observed that all synthesized systems provide overshoot values lower than those associated with the classical modulus optimum, while simultaneously achieving substantially faster response times. The first entrance into the admissible corridor occurs within approximately ($4.7T_v$) for all investigated astatism orders, demonstrating the effectiveness of the proposed optimization criterion.

To assess practical performance, simulations were performed using a milling-machine cutting-power control model. The system was subjected to a linearly increasing reference signal and abrupt load variations corresponding to instantaneous changes in cutting depth. A disturbance causing a 50% variation in cutting power was applied after 0.05 s of operation. The transient responses obtained for different astatism orders are shown in Fig. 6.

The simulation results reveal that increasing the astatism order improves disturbance rejection and reduces the steady-state tracking error. The smallest dynamic deviations were observed for the system with an astatism order of 1.5. However, the improvement in accuracy becomes progressively smaller as the order decreases below the recommended range, whereas the implementation complexity of the controller increases considerably. Therefore, practical considerations require balancing dynamic performance and controller complexity.

A more detailed analysis of the load-application and load-removal transients is presented in Fig. 7. The proposed control system effectively compensates for sudden changes in cutting conditions by automatically adjusting the feed rate. As a result, the amplitude of the cutting-power disturbance is reduced from approximately 200 W to only 19 W for ($\mu = 1.5$) and to about 39 W for the recommended value ($\mu = 1.7$). Furthermore, the transient duration remains below 0.02

s in all investigated cases, indicating excellent dynamic performance and rapid disturbance suppression.

An important property of systems with a fractional astatism order greater than unity is demonstrated in Fig. 8. Unlike conventional first-order astatism systems, which exhibit a constant velocity error under ramp reference signals, fractional-order systems provide a gradual reduction of the velocity error over time. This property results from the additional memory effects introduced by fractional integration operators. Consequently, the system becomes increasingly accurate during prolonged operation and is capable of tracking slowly varying technological references with higher precision.

At the beginning of the start-up process, the dynamic errors were found to be approximately two to four times larger than those observed at the final stage of the transient. This behavior is expected because the largest mismatch between the reference and controlled variables occurs during the initial acceleration period. As the transient evolves, the tracking error decreases significantly. Although the configuration with an astatism order of 1.5 demonstrated the highest accuracy, the overall differences in cutting-power stabilization performance among the investigated configurations were relatively small.

Since the controlled variable in the considered application is cutting power rather than position or speed, the ultimate selection criterion should not be based solely on tracking accuracy. The structural complexity of the controller, computational requirements, and implementation feasibility must also be considered. Taking these factors into account, astatism-order values in the range of 1.7–1.8 provide the most favorable compromise between dynamic accuracy, disturbance rejection capability, response speed, and controller simplicity.

Overall, the obtained results confirm the effectiveness of the proposed synthesis methodology. The combination of the newly developed optimization criterion, genetic-algorithm-based parameter tuning, and fractional-integral control structures makes it possible to achieve high-performance cutting-power stabilization with fast transient responses, low overshoot, and improved dynamic accuracy under variable technological operating conditions.

## V. CONCLUSIONS.

A methodology for the synthesis of closed-loop cutting-power stabilization systems for milling machines based on fractional-integral controllers with an increased astatism order has been developed. The proposed approach makes it possible to shape the desired dynamic characteristics of the control system and to ensure high-accuracy maintenance of cutting power under variable technological loading conditions.

A novel transient-response performance criterion has been substantiated. The proposed criterion takes into account not only the magnitude of the control error but also the extent to which the transient response exceeds the permissible deviation corridor. In contrast to conventional optimization criteria, the proposed approach provides fast transient responses with limited overshoot, which is particularly important for control systems used in machining processes.

The optimal parameters of closed-loop systems for different values of the fractional astatism order were determined using a genetic optimization algorithm. Relationships between the optimal transfer-function coefficients and the astatism order were obtained, and the parameter regions providing the best transient-response quality were identified.

Based on the optimization results, fractional-integral controller structures for the cutting-power stabilization system were synthesized. Analytical relationships were established that make it possible to determine controller parameters according to the characteristics of the controlled plant and the required control-performance specifications. The feasibility of implementing controllers containing fractional-integral elements for the realization of systems with an astatism order greater than unity was demonstrated.

The simulation results confirmed the high effectiveness of the proposed solutions. It was established that, for load variations causing a 50% change in cutting power, the synthesized system significantly reduces the amplitude of transient power deviations and provides rapid disturbance rejection. The transient-process duration does not exceed 0.02 s, indicating the high dynamic performance of the developed control system.

It was shown that the use of a fractional astatism order greater than one ensures a gradual reduction of the velocity-error component under a linearly varying reference signal and contributes to improved dynamic accuracy of the system. At the same time, an increase in the astatism order leads to a more complex controller structure, which should be taken into account during practical implementation.

The conducted research demonstrated that, for cutting-power stabilization problems, an astatism order in the range of 1.7–1.8 is the most appropriate choice, providing a rational compromise between control accuracy, system response speed, and the implementation complexity of the fractional-integral controller.

The practical significance of the obtained results lies in the possibility of applying them to the development of modern automatic control systems for metal-cutting machine tools and other technological installations requiring high-precision regulation of process energy parameters. The proposed methods contribute to improved stability of technological operating conditions, enhanced machining quality, and increased efficiency of industrial equipment operation.

CONFLICT OF INTEREST STATEMENT

**The authors declare no conflicts of interest regarding the publication of this paper.**



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